

# TEM BANDPASS FILTERS HAVING EIGHTH-WAVELENGTH COUPLED STUBS

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## Abstract

A simple technique is presented for design of TEM-mode bandpass filters that use eighth-wavelength coupled stubs in their realization. The filters, either of two dual forms, are identical in theoretical performance to conventional planar parallel-coupled filters often used in stripline fabrication; and to the interdigital filter, or the equivalent quarter-wavelength shorted stubs spaced at quarter-wavelength intervals along a transmission line. The design method uses existing tabulated element values and a novel Kuroda transformation which has been previously used in the design of directional filters.

## Introduction

Several papers<sup>1, 2</sup> have been published on the design of TEM-mode bandpass filters using a planar center conductor of parallel-coupled quarter-wavelength conductors. The open-circuited dual form of this filter type has been used many times in stripline packages; whereas the short-circuited form is sometimes used in high average power requirements to present a good thermal path to ground. The network for these filters is identical to that of an interdigital filter<sup>3</sup> or to the equivalent filter comprised of quarter-wavelength shorted stubs hung in shunt at quarter-wavelength intervals along a transmission line.

The new bandpass filter, which has eighth-wavelength coupled stubs in its realization, also has the same equivalent network. It is a somewhat different realization from the dual parallel-coupled types. The new filter also can be realized in either of two dual forms; one involving open-circuited coupled stubs, the other having short-circuited coupled stubs. The filter's topological form results from a novel Kuroda transformation which has been used in the design of directional filters<sup>4</sup>.

## The "Elemental" Network

Consider two parallel-coupled rods used as a two-port network with both ports at the same end. The identical coupled rods will support even and odd modes of characteristic impedance,  $Z_{oe}$  and  $Z_{oo}$ , and will be matched to a system impedance level,  $Z_o = \sqrt{(Z_{oe}Z_{oo})}$ . Now add an equal length unit element (u.e.) transmission line of impedance,  $Z_o$ , to one of the ports. If the open end of the coupled rods is left open-circuited, we refer to the structure as an "Even Mode Element," if this same end is short-circuited, it will be called an "Odd Mode Element" (Figure 1(a)). The common length of each line in such "elements" will be referred to as "half-length" to distinguish it from a "full-length" line, the latter of which will be quarter wavelength at the center frequency of the bandpass filter. The equivalent half-length distributed networks are shown in Figure 1(b) for even and odd mode cases, where it is implied that a capacitance (inductance) symbol denotes an open (short) half-length distributed line of characteristic admittance (impedance),  $Y_o(Z_o)$ . By applying Kuroda's Identities<sup>5</sup>, the half-length u.e. can be taken to the other side of the elemental network and the resultant L-C elements in the half-length frequency variable,  $\Omega' = \tan(\theta/2)$ , may be combined into FULL-LENGTH distributed L(C) elements in the variable,  $\Omega = \tan(\theta)$ , plus an

ideal transformer (Figure 1(c)). The two networks in Figure 1(c) are duals. We consider further only the odd mode geometry; results can be applied to the even mode case by dual relations. The half-length u.e. impedance remains unchanged in the transformation. The coupling coefficient,  $k$ , FULL-LENGTH normalized inductance,  $L/Z_o (=C/Y_o)$ , and transformer turns ratio,  $n$ , are interrelated by

$$k = \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} = \frac{2n}{1+n^2} = \frac{2\sqrt{1+Z_o/L}}{2+Z_o/L} \quad (1)$$

$$n = \frac{\sqrt{Z_{oe}} + \sqrt{Z_{oo}}}{\sqrt{Z_{oe}} - \sqrt{Z_{oo}}} = k^{-1} + \sqrt{k^{-2} - 1} = \sqrt{1+Z_o/L} \quad (2)$$

$$\left(\frac{C}{Y_o}\right) \frac{L}{Z_o} = \frac{1}{n^2 - 1} = \frac{1}{2\sqrt{k^{-2} - 1} (k^{-1} + \sqrt{k^{-2} - 1})} \quad (3)$$

## Odd-Mode Network and Interdigital Filters

The odd-mode network is comprised of elemental geometries like that of Figure 1(a) (odd) having the equivalent circuit of Figure 1(c) (odd). Two such identical circuits, back-to-back, can be cascaded to produce the overall network shown in Figure 2(a) for four sections. If each u.e. in the network is matched to unity (say, 50 ohms), and coupled sections at either end of each resulting FULL-LENGTH u.e. are identical, then each adjacent pair of ideal transformers will have equal turns ratios back-to-back, and will disappear. The resulting network of all FULL-LENGTH distributed elements (excepting the end-most half-length u.e.'s) is shown in Figure 2(b). Ignoring half-length u.e.'s at the ends, the distributed network of FULL-LENGTH L's and u.e.'s is identical in form to that of an interdigital filter<sup>3</sup>.

The interdigital filter of  $N$  resonators can be shown by a Kuroda Identity to contain only a single inductor contributing to the degree of polynomial representation. The remaining  $N-1$  contributions are from the u.e.'s. The odd (and even) mode filter under investigation is likewise composed, and its Chebyshev insertion loss function,  $|s_{21}|^{-2}$ , is given by<sup>6</sup>

$$|s_{21}|^{-2} = 1 + \epsilon^2 \left[ T_1 \left( \frac{\tan \theta_c}{\tan \theta} \right) T_{N-1} \left( \frac{\cos \theta}{\cos \theta_c} \right) - U_1 \left( \frac{\tan \theta_c}{\tan \theta} \right) U_{N-1} \left( \frac{\cos \theta}{\cos \theta_c} \right) \right]^2 \quad (4)$$

Tables of element values have been prepared<sup>7</sup> for symmetrical filters having the insertion loss function given in Equation (4).

#### Changing the Interior Impedance Level

The set of odd-mode filter element values (Figure 2(b)) is but one of an infinitude that can produce identical two-port responses dependent upon the interior impedance level. The physical geometry of an interdigital lends itself to easy visualization of the static-capacity array associated with the interdigital rods. Since the electrical network for the new filter (without half-length u.e.'s at each end) is identical to that of the interdigital filter, a capacitance array of inverted impedance values (Figure 2(c)) can be used to represent the FULL-LENGTH distributed portion of the odd-mode filter. A transformation performed on the capacitance matrix<sup>8</sup> by multiplying interior rows and corresponding columns by symmetrically-disposed admittance factors,  $N_i$ , changes the node impedances by  $N_i^{-2}$ . The resultant network has new u.e.

impedances (Figure 2(d)) and has double-primed inductance values given by (Note:  $N_o = N_1 = 1$ )

$$L_i'^{-1} = N_i \left[ -N_{i-1} + (2 + L_i'^{-1}) N_i - N_{i+1} \right] ; \quad (5)$$

$$i = 1, 2, \dots, \text{int}(N/2) .$$

The tabulated interdigital filter elements<sup>7</sup> were synthesized using an interior impedance criterion of equal impedance-to-ground from each node in the capacitance array. From this set any other set of equivalent elements can be obtained by an appropriate matrix transformation: one such transformation will result in unity values for the u.e.'s as in Figure 2(b).

#### Realizing the Eighth-Wavelength Coupled-Stub Filter

From the network of Figure 2(d), we wish to determine the coupling coefficients in the eighth-wavelength coupled-stub filter and the characteristic impedances of interconnecting lines. To accomplish this, we apply the equivalence of Figures 1(c) and 1(b) to each "elemental" half-section network. Each shunt inductor,  $L_i''$ , in Figure 2(d) is divided into two parallel shunt inductors,  $L_{2i-1}'''$  and  $L_{2i}'''$ . Each FULL-LENGTH u.e.,  $z_{i,i+1}'$ , is subdivided into a cascade of two identical half-length u.e.'s. The inductor values,  $L_{2i-1}'''$  and  $L_{2i}'''$ , are given in terms of  $L_i'$  and the node admittance factors,  $N_i$ , of Figure 2(c) by (Note:  $N_o = N_1 = 1$ )

$$L_{2i-1}''' = \left( \frac{N_{i-1} + N_{i+1}}{N_{i-1}} \right) L_i' ;$$

$$L_{2i}''' = \left( \frac{N_{i-1} + N_{i+1}}{N_{i+1}} \right) L_i' ; \quad (6)$$

$$i = 1, 2, \dots, \text{int}(N/2) .$$

in order that the back-to-back transformers cancel in Figure 2(e). Combining Equations (5) and (6), together with the fact that  $L_i' = L_i/2$ , gives

$$L_{2i-1}''' = \frac{(N_{i-1} + N_{i+1}) L_i}{N_{i-1} N_i [2(1 + L_i) N_i - (N_{i-1} + N_{i+1}) L_i]} ;$$

$$L_{2i}''' = \frac{(N_{i-1} + N_{i+1}) L_i}{N_i N_{i+1} [2(1 + L_i) N_i - (N_{i-1} + N_{i+1}) L_i]} ;$$

$$= \frac{N_{i-1}}{N_{i+1}} L_{2i-1}''' ; \quad (7)$$

$$i = 1, 2, \dots, \text{int}(N/2) .$$

Note that if all  $N_i = 1$ , then  $L_{2i-1}''' = L_{2i}''' = L_i$  as in Figure 2(a). Further, the ratio of inductors,  $L_{2i-1}'''$ ,  $L_{2i}'''$ , to their respective adjacent u.e. impedances is

$$\frac{L_{2i-1}'''}{N_{i-1}^{-1} N_i^{-1}} = \frac{L_{2i}'''}{N_i^{-1} N_{i+1}^{-1}} ;$$

$$= \frac{(N_{i-1} + N_{i+1}) L_i}{2N_i (1 + L_i) - (N_{i-1} + N_{i+1}) L_i} \quad (8)$$

or, cf (3),

$$n_i'^2 = \frac{2N_i (1 + L_i)}{(N_{i-1} + N_{i+1}) L_i} . \quad (9)$$

The following results have been accomplished: through the admittance transformation,  $N_i$ , each adjacent pair of half-length elements of odd form in Figure 1(a) has been transformed into another pair having different half-length u.e. characteristic impedances given by

$$z_{2i-1}' = N_{i-1}^{-1} N_i^{-1} ; \quad z_{2i}' = N_i^{-1} N_{i+1}^{-1} ;$$

$$i = 1, 2, \dots, \text{int}(N/2) , \quad (10)$$

respectively, but having equal (but different from the original) coupling coefficients obtained by placing  $n_i'$  of (9), or  $L/Z_o$  of (8), into (1) to give the new set of coefficients,  $k_i'$ . In general, a reduced set of interior admittance levels ( $N_i < 1$ ) will provide smaller values of coupling ( $k_i' < k_i$ ), which will ease the coupling problem required for equiripple response in the filter.

Each "elemental" half-length section in Figure 2(e) is "realized" through the inverse process of Figures 1(c) to 1(b). The resulting filter will take the form shown in Figure 3(a). Except for the unit-value half-length u.e.'s at each end, the remaining FULL-LENGTH u.e.'s may have different impedances, and each contains two cascaded coupled half-length sections at its center. The latter two sections can be combined into a single coupled-stub section having smaller coupling coefficient than either constituent. If the two constituent coefficients are  $k_i$  and  $k_j$ , then it can be shown<sup>4</sup> that the combined coupling coefficient,  $k$ , is given by

$$k = \frac{k_i k_j}{1 + \sqrt{1 - k_i^2} \sqrt{1 - k_j^2}} \quad (11)$$

with  $k < k_i$  and  $k < k_j$ . The final odd-mode bandpass filter having eighth-wavelength short-circuited coupled stubs is shown in Figure 3(b). The dual even mode filter with open-circuited coupled stubs is shown in Figure 3(c). In the latter network, if each characteristic admittance is made equal to the corresponding characteristic impedance in the odd-mode network of Figure 3(b), their amplitude responses will be identical.

#### Summary and Conclusions

New TEM mode dual filter forms, each having coupled eighth-wavelength stubs imbedded in the center of cascaded quarter-wavelength transmission lines, have been devised and a method presented for achieving optimum equiripple response from tabulated element values. The new even mode filter is likely to find most application in stripline requirements; whereas, the odd mode form may prove useful in high average power requirements.

#### References

1. G. L. Matthaei, et al, Microwave Filters, Impedance Matching Networks, and Coupling Structures, New York: McGraw-Hill, 1964.
2. S. B. Cohn, "Parallel-Coupled Transmission Line Resonator Filters," IRE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, MTT-6, April, 1958; pp. 223-231.
3. R. J. Wenzel, "Exact Theory of Interdigital Bandpass Filters and Related Coupled Structures," IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, MTT-13, No. 5, September, 1965; pp. 559-575.
4. J. L. B. Walker, "Exact and Approximate Synthesis of TEM Mode Transmission-Type Directional Filters," accepted November, 1976 for publication in IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES.
5. R. J. Wenzel, "Exact Design of TEM Micro-wave Networks Using Quarter-Wave Lines," IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, MTT-12, No. 1, January, 1964; p. 98.
6. M. C. Horton and R. J. Wenzel, "General Theory and Design of Optimum Quarter-Wave TEM Filters," IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, MTT-13, No. 3, May, 1965; p. 319.
7. Wenzel, op. cit., footnote 9, p. 570.
8. R. J. Wenzel, "Theoretical and Practical Applications of Capacitance Matrix Transformations to TEM Network Design," IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, MTT-14, No. 12, December, 1966; pp. 635-647.

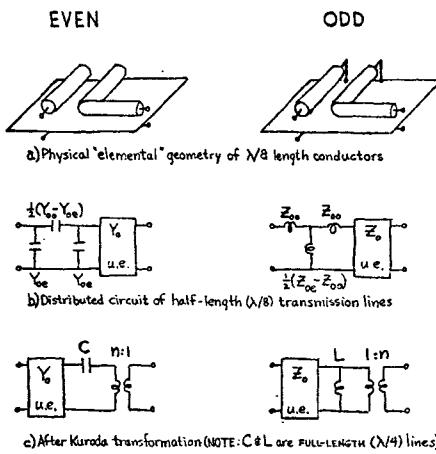


FIGURE 1 EVEN and ODD MODE ELEMENTAL SECTIONS

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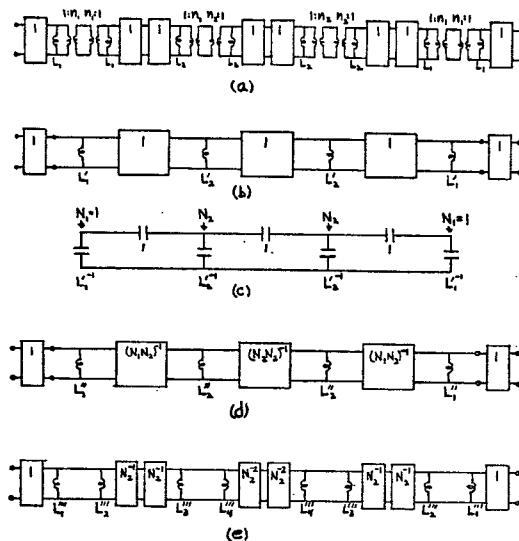


FIGURE 2 STEPS IN DESIGN OF N=4 FILTER

- Starting with matched elemental half-length sections
- Equivalent interior FULL-LENGTH network
- Capacitance array for interior network
- FULL-LENGTH network after capacitance transformation
- Resultant array of elemental sections

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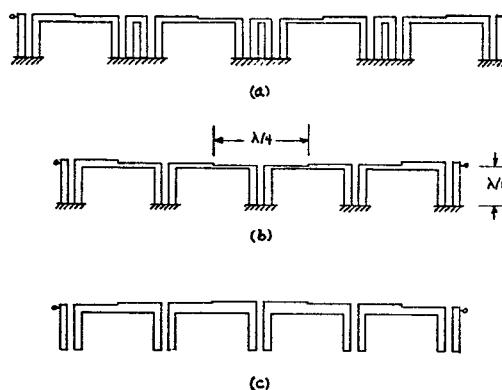


FIGURE 3 COUPLED EIGHTH-WAVELENGTH FILTERS

- Odd-mode physical realization from Fig. 2e
- Odd-mode final physical filter form
- Even-mode final physical filter form

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